XUSUSIY HOSILALI DIFFERENSIAL TENGLAMALARNI KANONIK KO‘RINISHGA KELTIRISH VA TIPINI ANIQLASHNING BAZI BIR USULLARI

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Annotasiya: Kanonik shaklini ta’minlovchi birinchi tartibli xususiy hosilali differensial tenglamaning yechimga ega bo’lish masalasi tenglamaning xarakteristik tenglamasi deb ataluvchi oddiy differensial tenglamaning umumiy integrali bilan uzviy bog’liqligi ko’riladi. oddiy differensial tenglamaning umumiy integrali bo’lishi zarur va yetarlik sharti bajarilishi isboti bilan beriladi. Xususiy hosilali differensial tenglamalarni 2 xil usul yordamida kanonik ko’rinishga keltirish va tipini aniqlash kabi masalalarni ko’rib chiqiladi.

Kalit so’zlar: Xususiy hosilali differensial tenglama, kanonik shakli, giperbolik tipi, parabolik tipi, elliptik tipi.

METHODS OF CANONICALIZATION OF PARTIAL DIFFERENTIAL EQUATIONS AND TYPE DETERMINATION

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Abstract: It is seen that the problem of having a solution of the first-order differential equation providing the canonical form is inextricably linked with the general integral of the ordinary differential equation, which is called the characteristic equation of the equation. is given by proving that the general integral of the ordinary differential equation is a necessary and sufficient condition. Issues

such as canonicalization of partial differential equations using 2 different methods and determining their type are considered.

Key words: differential equation with particular derivative, canonical form, hyperbolic type, parabolic type, elliptic type.

Barchamizga ma'lumki xususiy hosilali defferensial tenglamalarda bir, ikki va undan ko'p o'zgaruvchilar bilan berilgan bo'lib bu masalani kanonik ko'rinishga keltirish usullarining bir nechtasidan foydalanib ikki va uch o'zgaruvchili differensial tenglamalarni keltirish masalasini ko'rib chiqamiz, bunda 2 xil ko'rinishdagi misollarni qaraylik.

$$u_{xx} + 8u_{xy} + 7u_{yy} + 3u_x - 2u_y + u + x = 0$$

va $$2u_{xx} + 2u_{xy} - 2u_{xz} + u_{yy} + 2u_{zz} = 0$$

Berilgan birinchi misolimizni quyidagicha usul yordamida yechaylik.

$$u_{xx} + 8u_{xy} + 7u_{yy} + 3u_x - 2u_y + u + x = 0$$

Bu misolimiz ikkinchi tartibli ikki o'zgaruvchili xususiy hosilali chiziqli tenglama bo'lib, bu misolni yechishdan avval yechish usuli haqida ma'lumotlar berib o'tamiz. Bu misolimiz umumiy holda quyidagi ko'rinishda yoziladi:

$$a_{11}(x,y)u_{xx} + 2a_{12}(x,y)u_{xy} + a_{22}(x,y)u_{yy} + b_1(x,y)u_x + b_2(x,y)u_y + c(x,y)u + f(x,y) = 0 \quad (2)$$

Agar (2) tenglamada erkli o'zgaruvchilarni o'zaro bir qiymatli

$$\xi = \varphi(x,y), \qquad \eta = \psi(x,y) \quad (3)$$

almashtirish bajarsak (2) differensial tenglamaga ekvivalent tenglamani hosil qilamiz. Ushbu yangi o'zgaruvchilarda (2) tenglamada ishtirok etayotgan xususiy hosilalarni hisoblaymiz:

$$\left.\begin{aligned}
&u_x = u_\xi \xi_x + u_\eta \eta_x \\
&u_y = u_\xi \xi_y + u_\eta \eta_y \\
&u_{xx} = u_{\xi\xi}\xi_x^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}\eta_x^2 + u_\xi \xi_{xx} + u_\eta \eta_{xx} \\
&u_{xy} = u_{\xi\xi}\xi_x\xi_y + u_{\xi\eta}(\xi_x\eta_y + \xi_y\eta_x) + u_{\eta\eta}\eta_x\eta_y + u_\xi \xi_{xy} + u_\eta \eta_{xy} \\
&u_{yy} = u_{\xi\xi}\xi_y^2 + 2u_{\xi\eta}\xi_y\eta_y + u_{\eta\eta}\eta_y^2 + u_\xi \xi_{yy} + u_\eta \eta_{yy}
\end{aligned}\right\} \quad (4)$$

(4) dagi ifodalarni (2) ga qo'yamiz va bir xil xususiy hosilalarni jamlab, (2) ga ekvivalent bo'lgan quyidagi xususiy hosilali differensial tenglamaga kelamiz:

$$\bar{a}_{11}(\xi,\eta)u_{\xi\xi}+2\bar{a}_{12}(\xi,\eta)u_{\xi\eta}+\bar{a}_{22}(\xi,\eta)u_{\eta\eta}+\bar{b}_1(\xi,\eta)u_\xi+\bar{b}_2(\xi,\eta)u_\eta+\bar{c}(\xi,\eta)u+\bar{f}(\xi,\eta)=0\,. \quad (5)$$

Bunda koeffisientlardagi funksiyalar (2) tenglama koeffisientlari orqali quyidagicha ifodalanadi

$$\left.\begin{aligned}
\bar{a}_{11}&=a_{11}\xi_x^2+2a_{12}\xi_x\xi_y+a_{22}\xi_y^2\\
\bar{a}_{12}&=a_{11}\xi_x\eta_x+a_{12}(\xi_x\eta_y+\xi_y\eta_x)+a_{22}\xi_y\eta_y\\
\bar{a}_{22}&=a_{11}\xi_y^2+2a_{12}\eta_x\eta_y+a_{22}\eta_y^2\\
\bar{b}_1&=a_{11}\xi_{xx}+2a_{12}\xi_{xy}+a_{22}\xi_{yy}+b_1\xi_x+b_2\xi_y\\
\bar{b}_2&=a_{11}\eta_{xx}+2a_{12}\eta_{xy}+a_{22}\eta_{yy}+b_1\eta_x+b_2\eta_y
\end{aligned}\right\} \quad (6)$$

Demak, o’zaro bir qiymatli akslantirishlar natijasida xususiy hosilali chiziqli differensial tenglama yana chiziqli differensial tenglamaga o’tar ekan. (6) dan ko’rinib turibdiki, agar biror $z=\varphi(x,y)$ funksiya

$$a_{11}z_x^2+2a_{12}z_xz_y+a_{22}z_y^2=0 \quad (7)$$

1-tartibli xususiy hosilali differensial tenglamaning yechimi bo’lsa, u holda (6) da $\xi=\varphi(x,y)$ deb olinsa $\bar{a}_{11}=0$ bo’ladi. Xuddi shu kabi mulohazalarni $\bar{a}_{12}$ va $\bar{a}_{22}$ koeffisientlar uchun ham aytish mumkin. Demak yangi o’zgaruvchilarni (5) differensial tenglamaning yuqori tartibli xususiy hosilalaridan ba’zilari nolga teng bo’ladigan qilib tanlash masalasi (7) birinchi tartibli xususiy hosilali differensial tenglamaning yechimini topish bilan uzviy bog’liq ekan. 2-tartibli xususiy hosilali differensial tenglamaning aralash ikkinchi tartibli xususiy hosilalari qatnashmagan bu sodda shakli odatda uning kanonik shakli deb yuritiladi.

Kanonik shaklini ta’minlovchi (7) birinchi tartibli xusuiy hosilali differensial tenglamaning yechimga ega bo’lish masalasi (2) tenglamaning xarakteristik tenglamasi deb ataluvchi

$$a_{11}dy^2-2a_{12}dydx+a_{22}dx^2=0 \quad (8)$$

oddiy differensial tenglamaning umumiy integrali bilan uzviy bog’liq bo’ladi. Uning umumiy integrallariga odatda (2) tenglamaning xarakteristik chiziqlari deb yuritiladi. Yuqoridagi tasdiqni biz quyidagi lemmada keltiramiz.

Lemma. $z=\varphi(x,y)$ funksiya (7) birinchi tartibli xususiy hosilali tenglamaning aynan o’zgarmasdan farqli yechimi bo’lishi uchun

$\varphi(x, y) = C,\ C = const$ ning (8) oddiy differensial tenglamaning umumiy integrali bo'lishi zarur va yetarlidir.

(8) oddiy differensial tenglama quyidagi ikki oddiy differensial tenglamaga ajraladi:

$$\left.\begin{aligned} \frac{dy}{dx} &= \frac{a_{12} - \sqrt{a_{12}^2 - a_{11}a_{22}}}{a_{11}} \\ \frac{dy}{dx} &= \frac{a_{12} + \sqrt{a_{12}^2 - a_{11}a_{22}}}{a_{11}} \end{aligned}\right\} \qquad (11)$$

(11) dagi ildiz belgisi ostidagi ifodaning qaralayotgan nuqtadagi qiymatiga qarab (2) tenglama quyidagi 3 tipga ajraladi.

Ta'rif. 1) Agar berilgan (x_0, y_0) nuqtada $\Delta = a_{12}^2(x_0, y_0) - a_{11}(x_0, y_0)a_{22}(x_0, y_0) > 0$ bo'lsa (2) tenglama bu nuqtada giperbolik tipli deyiladi.

2) Agar berilgan (x_0, y_0) nuqtada $\Delta = a_{12}^2(x_0, y_0) - a_{11}(x_0, y_0)a_{22}(x_0, y_0) = 0$ bo'lsa (2) tenglama bu nuqtada parabolik tipli deyiladi.

3) Agar berilgan (x_0, y_0) nuqtada $\Delta = a_{12}^2(x_0, y_0) - a_{11}(x_0, y_0)a_{22}(x_0, y_0) < 0$ bo'lsa (2) tenglama bu nuqtada elliptik tipli deyiladi.

(2) tenglamaning hamma giperbolik tipli bo'ladigan nuqtalari to'plami shu tenglamaning giperboliklik to'plami, parabolik tipli nuqtalari to'plami parabolik sohasi va elliptik tipli bo'ladigan nuqtalari to'plami uning elliptiklik sohasi deyiladi. Agar (2) tenglama qaralayotgan soha nuqtalarida bir nechta tipga ega bo'lsa, bu sohada tenglama aralash tipli deyiladi.

Endi (2) tenglama faqat bir tipga ega bo'ladigan biror D to'plamni qaraymiz. (11) ga asosan bu sohaning har bir nuqtasidan (2) tenglamaning ikkita xarakteristik chizig'I o'tadi. Xususan, (2) tenglama D sohada giperbolik tipli bo'lganda ikkala turli haqiqiy qiymatli, parabolik holda ustma-ust tushuvchi haqiqiy qiymatli va elliptik bo'lganda esa ikkita qo'shma kompleks qiymatli xarakteristik chiziqlar hosil bo'ladi. (2) tenglamaning kanonik shaklini topish uchun bu hollarni alohida-alohida qarab chiqamiz.

1) D sohada (2) giperbolik tipli bo'lsin, ya'ni uning barcha nuqtalarida $\Delta = a_{12}^2 - a_{11}a_{22} > 0$ tengsizlik o'rinli. Bu holda (11) ning har ikkala tenglamasi haqiqiy qiymatli

$$\varphi(x, y) = C_1,\ \psi(x, y) = C_2$$

umumiy integrallarga ega bo'ladi. Mavzu boshida aytilgan yangi o'zgaruvchilarni $\xi = \varphi(x, y), \quad \eta = \psi(x, y)$

kabi tanlaymiz. U holda Lemma va (6) ga asosan $\bar{a}_{11} = \bar{a}_{22} = 0, \bar{a}_{12} \neq 0$ bo'lib, yangi o'zgaruvchilarda (2) tenglama quyidagi ko'rinishni oladi:

$$u_{\xi\eta} = F_1(\xi, \eta, u, u_\xi, u_\eta), \qquad (12)$$

bunda

$$F_1(\xi, \eta, u, u_\xi, u_\eta) = -\frac{\bar{b}_1(\xi, \eta)}{2\bar{a}_{12}(\xi, \eta)} u_\xi - \frac{\bar{b}_2(\xi, \eta)}{2\bar{a}_{12}(\xi, \eta)} u_\eta - \frac{\bar{c}(\xi, \eta)}{2\bar{a}_{12}(\xi, \eta)} u - \frac{\bar{f}(\xi, \eta)}{2\bar{a}_{12}(\xi, \eta)}.$$

Odatda (12) tengalamaga giperbolik tenglamalarning 1-tur kanonik shakli deyiladi.

Agar unda $\alpha = \dfrac{\xi + \eta}{2}, \quad \beta = \dfrac{\xi - \eta}{2}$ almashtirishlarni bajarsak

$$u_{\xi\eta} = \frac{1}{4}(u_{\alpha\alpha} - u_{\beta\beta})$$

bo'lib, (12) ga asosan giperboik tenglamalarning 2-tur kanonik shakli

$$u_{\alpha\alpha} - u_{\beta\beta} = 4F_1(\alpha, \beta, u, u_\alpha, u_\beta)$$

hosil bo'ladi.

2) D sohada (2) parabolik tipli bo'lsin, ya'ni uning barcha nuqtalarida $\Delta = a_{12}^2 - a_{11}a_{22} = 0$ tenglik o'rinli. Bu holda (11) ning har ikkala tenglamasi bitta haqiqiy qiymatli

$$\varphi(x, y) = C$$

umumiy integralga ega bo'ladi. Bu holda yangi o'zgaruvchilarni

$$\xi = \varphi(x, y), \quad \eta = \psi(x, y)$$

kabi tanlaymiz. Bunda $\psi(x, y)$ orqali $\varphi(x, y)$ bilan chizqli bogl'anmagan ixtiyoriy funksiya tanlangan. U holda Lemma va (6) ga asosan $\bar{a}_{11} = 0, \bar{a}_{12} \neq 0$ va bo'lib, $a_{12} = \sqrt{a_{11}}\ \sqrt{a_{22}}$ bo'lganligi uchun (6) dan

$$\bar{a}_{12} = (\sqrt{a_{11}}\xi_x + \sqrt{a_{22}}\xi_y)(\sqrt{a_{11}}\eta_x + \sqrt{a_{22}}\eta_y) = 0$$

ekanligini olamiz. Natijada (5) da $\bar{a}_{22}$ bo'lish bilan giperbolik tipli tenglamalarning kanonik shakli ni hosil qilamiz:

$$u_{\eta\eta} = F_2(\xi,\eta,u,u_\xi,u_\eta).$$

Bunda

$$F_1(\xi,\eta,u,u_\xi,u_\eta) = -\frac{\bar{b}_1(\xi,\eta)}{\bar{a}_{22}(\xi,\eta)}u_\xi - \frac{\bar{b}_2(\xi,\eta)}{\bar{a}_{22}(\xi,\eta)}u_\eta - \frac{\bar{c}(\xi,\eta)}{\bar{a}_{22}(\xi,\eta)}u - \frac{\bar{f}(\xi,\eta)}{\bar{a}_{22}(\xi,\eta)}.$$

3) D sohada (2) tenglama elliptik tipli bo'lsin, ya'ni uning barcha nuqtalarida $\Delta = a_{12}^2 - a_{11}a_{22} < 0$ tengsizlik o'rinli. Bu holda (11) ikkita qo'shma kompleks umumiy integrallarga ega bo'ladi

$$\varphi(x,y) + i\psi(x,y) = C_1,\ \varphi(x,y) - i\psi(x,y) = C_2.$$

Demak, $u_{xx} + 8u_{xy}$+$7u_{yy}$ +$3u_x - 2u_y + u + x = 0$

$a_{11} = 1,\ a_{12} = 4,\ a_{22} = 7$ tenglama koeffisiyentlari. $\Delta = a^2{}_{12} - a_{11}a_{22}$ ifodaning kiymatini hisoblaymiz. $\Delta = 9 > 0$, demak tenglama giperbolik tipga tegishli. (9) xarakteristik tenglamani yechamiz. $\frac{dy}{dx}$=1$\Rightarrow$ x-y=c, $\frac{dy}{dx}$=7 $\Rightarrow$ 7x-y=c

Umumiy integrallardan birini ξ va ikkinchisini η bilan quyidagicha ξ=x-y, η=7x-y belgilab, (4) formulaga qo'yamiz

$$u_x = u_\xi + 7u_\eta$$

$$u_y = -u_\xi - u_\eta$$

$$u_{xx} = u_{\xi\xi} + 14u_{\xi\eta} + 49u_{\eta\eta}$$

$$u_{xy} = -u_{\xi\xi} - 8u_{\xi\eta} - 7u_{\eta\eta}$$

$$u_{yy} = u_{\xi\xi} + 2u_{\xi\eta} + u_{\eta\eta}$$

Topilgan ifodalarni yuqoridagi tenglamaga olib borib qo'yamiz.

$u_{\xi\xi} + 14u_{\xi\eta} + 49u_{\eta\eta} - 8u_{\xi\xi} - 64u_{\xi\eta} - 56u_{\eta\eta} + 7u_{\xi\xi} + 14 + 7u_{\eta\eta} + 3u_\xi + 21u_\eta + 2u_\xi + 2u_\eta + u + x = 0$,

soddalashtirsak

$$-50u_{\xi\eta} + 23u_\eta + 5u_\xi + u + x = 0$$

Bu yerda (12) ga ko'ra

$$F_1(\xi,\eta,u,u_\xi,u_\eta) = (23u_\eta + 5u_\xi + u + x)$$

Belgilasak

$$50u_{\xi\eta} = F_1(\xi,\eta,u,u_\xi,u_\eta)$$

Demak, bu tenglamamiz giperbolik tenglamalarning 1-*tur kanonik shakli* deyiladi.

Endi bu

$$2u_{xx} + 2u_{xy} - 2u_{xz} + u_{yy} + 2u_{zz} = 0$$

Demak tenglamamizni quyidagicha umumiy ko'rinishda yozishimiz mumkin

$$\sum_{i=1}^{n} \mu_k u_{y_k y_k} + \bar{\phi}(y,u,u_{y_1},\dots,u_{y_n}) = 0 \qquad (13)$$

Ikkinchi tartibli differensial tenglamaning aralash hosilalar qatnashmagan (13) ko'rinishi, odatda uning kanonik ko'rinishi deyiladi.

Agar barcha $\mu_k = 1$ yoki barcha $\mu_k = -1$, $k = 1,2,\dots,n$ bo'lsa, ya'ni Q forma mos ravishda musbat yoki manfiy aniqlangan (definit) bo'lsa, berilgan tenglama $x \in \Omega$ nuqta elliptik tipdagi yoki elliptik tenglama deyiladi.

Agar μ_k koeffitsientlardan bittasi manfiy, qolganlari musbat (yoki aksincha) bo'lsa, berilgan tenglama $x \in \Omega$ nuqtada giperbolik tenglama deyiladi.

Agar μ_k koeffitsientlardan bittasi nolga teng, qolganlari noldan farqli va bir xil ishorali bo'lsa berilgan tenglama $x \in \Omega$ nuqtada parabolik tenglama deyiladi.

Ω sohaning turli qismida berilgan tenglama xar har xil tipga tegishli bo'lsa, bunday tenglamaga aralash tipdagi tenglama deyiladi.

Misolimizni yechishda quyidagi $\lambda_1 = u_x$, $\lambda_2 = u_y$, $\lambda_3 = u_z$ belgilashlardan foydalanaylik.

$$Q(\lambda_1,\lambda_2,\lambda_3) = 2{\lambda_1}^2 + 2\lambda_1\lambda_2 - 2\lambda_1\lambda_3 + {\lambda_2}^2 + 2{\lambda_3}^2 =$$

$$= (\lambda_1+\lambda_2)^2 + (\lambda_1-\lambda_3)^2 + {\lambda_3}^2$$

$$\xi_1 = (\lambda_1+\lambda_2)\,,\ \xi_2 = (\lambda_1-\lambda_3),\ \xi_3 = \lambda_3$$

ya'ni

$$\lambda_1 = \xi_2 + \xi_3\,,\ \lambda_2 = \xi_1 - \xi_3 - \xi_2, \quad \lambda_3 = \xi_3$$

almashtirishdan foydalansak,

Q formani kanonik ko‘rinishga keltiradi: $Q(\lambda_1,\lambda_2,\lambda_3)={\xi_1}^2+{\xi_2}^2+{\xi_3}^2$.

Shunday qilib,

$$M=\begin{pmatrix}0 & 1 & 1\\ 1 & -1 & -1\\ 0 & 0 & 1\end{pmatrix}$$

M matrisali quyidagi xosmas affin almashtirishlari

$$\lambda_1=\xi_2+\xi_3\ ,\quad \lambda_2=\xi_1-\xi_3-\xi_2,\quad \lambda_3=\xi_3$$

Berilgan differensial tenglamani kanonik ko‘rinishga keltiradigan xosmas affin almashtirishining matrisasi M matrisaga simmetrik bo‘lgan matrisa bo‘ladi:

$$M^*=\begin{pmatrix}0 & 1 & 0\\ 1 & -1 & 0\\ 1 & -1 & 1\end{pmatrix}$$

bu almashtirish quyidagi ko‘rinishga ega: $\xi=y,\quad \eta=x-y,\quad \zeta=x-y+z$

$$u_{xx}=u_{\zeta\zeta}+2u_{\zeta\eta}+u_{\eta\eta}$$

$$u_{xy}=u_{\xi\eta}-u_{\eta\eta}-u_{\zeta\zeta}-2u_{\zeta\eta}+u_{\xi\zeta}$$

$$u_{zy}=-u_{\zeta\zeta}-u_{\zeta\eta}+u_{\zeta\xi}$$

$$u_{xz}=u_{\zeta\zeta}+u_{\eta\zeta}$$

$$u_{yy}=u_{\xi\xi}-2u_{\xi\eta}+u_{\eta\eta}+u_{\zeta\zeta}+2u_{\zeta\eta}-2u_{\xi\zeta}$$

$$u_{zz}=u_{\zeta\zeta}$$

Bularni

$$2u_{xx}+2u_{xy}-2u_{xz}+u_{yy}+2u_{zz}=0$$

tenglamaga olib borib qo’ysak,

$$2u_{\zeta\zeta}+4u_{\zeta\eta}+2u_{\eta\eta}+2u_{\xi\eta}-2u_{\eta\eta}-2u_{\zeta\zeta}-4u_{\zeta\eta}+2u_{\xi\zeta}-2u_{\zeta\zeta}-2u_{\eta\zeta}+$$
$$u_{\xi\xi}-2u_{\xi\eta}+u_{\eta\eta}+u_{\zeta\zeta}+2u_{\zeta\eta}-2u_{\xi\zeta}+2u_{\zeta\zeta}=0$$

Quyidagi

$$u_{\xi\xi}+u_{\zeta\zeta}+u_{\eta\eta}=0$$

Kanonik ko’rinishga ega bo’lamiz.

Bundan xulosa qilish mumkunki, berilgan tenglamamiz elliptik tipdaga tegishli ekan.

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